Unit 17
Day 1
Natural Logs

Reminder . . .

Theorem on Logarithmic Inverses

For $a > 0$, $a \neq 1$:

$$a^{\log_a x} = x \quad \text{and} \quad \log_a\left(a^x\right) = x$$

Note - could have been part of Unit 16 Day 3 notes

VALUE OF e

To nine decimal places,

$$e \approx 2.718281828$$

Discovering the Number e through compound interest

Ignoring the principal, the interest rate, and the number of years by setting all these variables equal to "1", and looking only at the influence of the number of compoundings, we get:

how often compounded	computation
yearly	$\boxed{\left(1+\frac{1}{1}\right)^1} = 2$
semi-annually	$\left(1+\frac{1}{2}\right)^2 = 2.25$
quarterly	$\left(1+\frac{1}{4}\right)^4 = 2.44140625$
monthly	$\left(1+\frac{1}{12}\right)^{12} \approx 2.61303529022\ldots$
weekly	$\left(1+\frac{1}{52}\right)^{52} \approx 2.69259695444\ldots$
daily	$\left(1+\frac{1}{365}\right)^{365} \approx 2.71456748202\ldots$
hourly	$\left(1+\frac{1}{8760}\right)^{8760} \approx 2.71812669063\ldots$
every minute	$\left(1+\frac{1}{525600}\right)^{525600} \approx 2.7182792154\ldots$
every second	$\left(1+\frac{1}{31536000}\right)^{31536000} \approx 2.71828247254\ldots$

In the world of finance, this is called a Future Value Interest Factor (FVIF)

Notice - the FVIF approaches 2.71828 . . . It approaches e

Also notice, compounding daily is not much worse than compounding continuously!

Some other "real-life" examples of the number e:

- population growth
- radioactive decay
- charge on a capacitor
- Newton's law of cooling (thermodynamics)
- plane waves (electrodynamics)
- Boltzmann Factor (thermodynamics)

Euler showed that

$$e = 1 + {}^{1}/_{1!} + {}^{1}/_{2!} + {}^{1}/_{3!} + \dots$$

and

$$\frac{e-1}{2} = \cfrac{1}{1 + \cfrac{1}{6 + \cfrac{1}{10 + \cfrac{1}{14 + \cfrac{1}{18 + \dots}}}}}$$

and

$$e - 1 = 1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{6 + \dots}}}}}}}}$$

He never would have done that if he had an Xbox! Or a cell phone for that matter!

$$f(x) = e^x$$

Find:

1) $f(\ln 7) = e^{\ln 7}$

$= 7$

2) $f\left(\ln\left(\frac{2}{e}\right)\right) =$

$e^{\ln \frac{2}{e}}$

$\frac{2}{e}$

$f(x) = \ln x$

1) $f(e^{\ln 5}) =$

$\ln e^{\ln 5}$

$\ln 5$

2) $f\left(e^{4\ln 2}\right)$

$\ln e^{4\ln 2}$

$\ln e^{\ln 2^4}$

$\ln 16$

Look at some of the problems from p. 371 -3 together.

Homework

p. 372-373: 71 , 73 , 74
p. 392: 43-52
p. 402: 1 -4